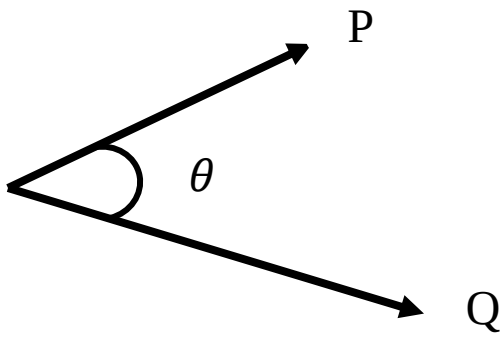
3.3 Dot product

The dot product of two vectors P and Q (in figure below) is defined as the product of their magnitudes times the cosine of the angle between them.

$$\vec{P} \cdot \vec{Q} = |P| \cdot |Q| \cos\theta$$

$$\vec{P} \cdot \vec{Q} = \vec{Q} \cdot \vec{P}$$



θ is the smaller angle from the first vector to the second vector. $(0 \le \theta \le 180)$

Note:-

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

i . i = 1	i . j = 0
j . j = 1	j . k = 0
k . k = 1	i . k = 0

$$\vec{A} \cdot \vec{B} = (\vec{A}_{xi} + \vec{A}_{yj} + \vec{A}_{zk})(\vec{B}_{xi} + \vec{B}_{yj} + \vec{B}_{zk})$$

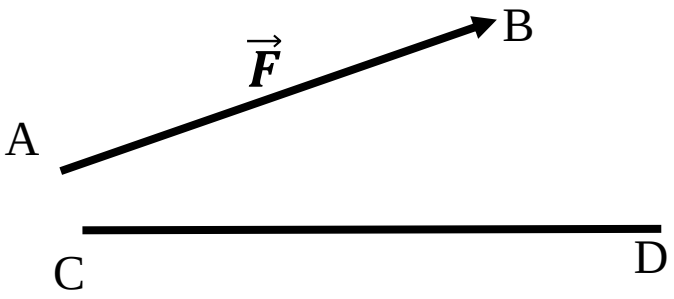
$$\vec{A} \cdot \vec{B} = (A_x B_x) + (A_y B_y) + (A_z B_z)$$

1

Projection (component) of force on line

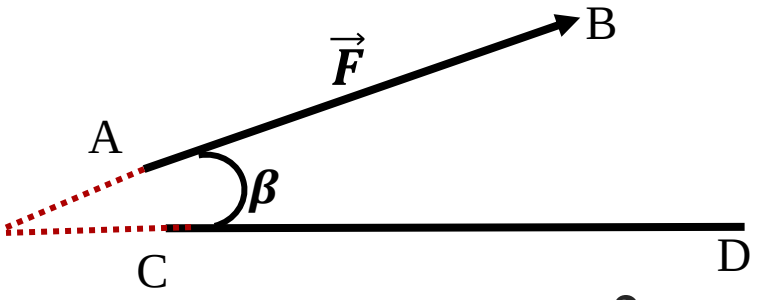
Let $\boldsymbol{F_{AB}}$ a force and CD is a line. The projection or (component) force $(\vec{F}_{AB})$ on line CD

Proj. $\boldsymbol{F_{AB}} = \vec{F}_{AB} \cdot \vec{n}_{CD}$



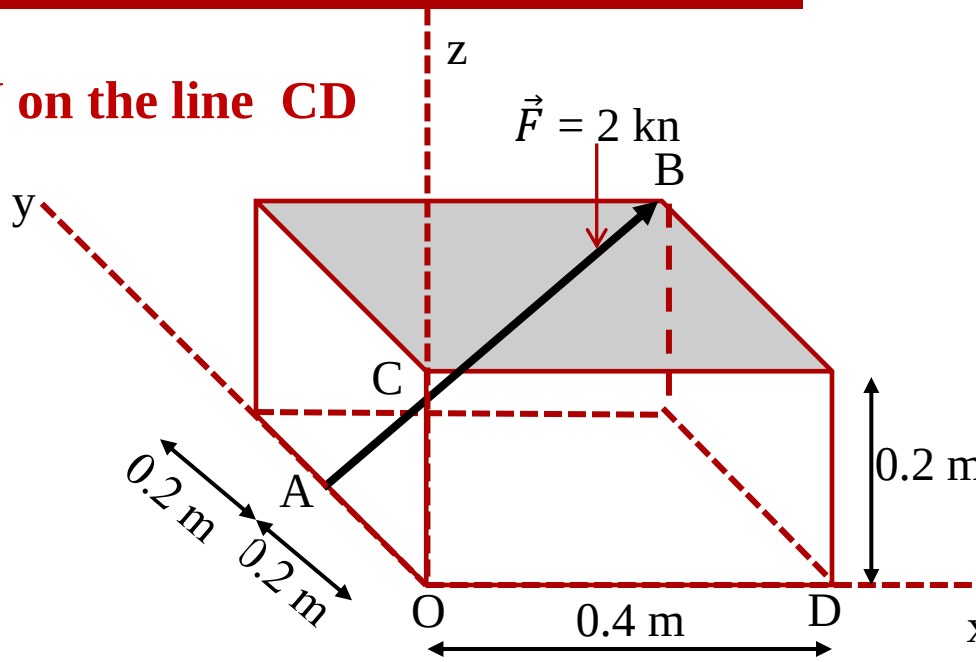
To find the angle between fore $\mathbf{F_{AB}}$ and line **CD**

Proj. $\boldsymbol{F_{AB}} = F_{AB} \cdot |n_{CD}| \cdot \cos\beta$



2

EX:- find the projection of force 2 KN on the line CD



Solution:

A (0, 0.2, 0), B (0.4, 0.4, 0.2)
C (0, 0, 0.2), D (0.4, 0, 0)

Proj F_{AB} on CD $= \vec{F}_{AB} \cdot \vec{n}_{CD}$

$F_{AB} = F_{AB} \cdot \vec{n}_{AB}$

$\vec{F}_{AB} = 2.\left(\frac{0.4i + 0.2j + 0.2k}{\sqrt{(0.4)^2+(0.2)^2+(0.2)^2}}\right) =$ 2. (0.816i + 0.408j + 0.408k)

$\vec{n}_{CD} = \frac{0.4i - 0.2k}{\sqrt{(0.4)^2+(0.2)^2}} =$ 0.895i − 0.447k

Proj F_{AB} on CD $= \vec{F}_{AB} \cdot \vec{n}_{CD}$

(1.632i + 0.816j + 0.816k)(0.895i − 0.447k) = 1.632 * 0.895 + 0.816*(−0.447)
= 1.0958 kn.

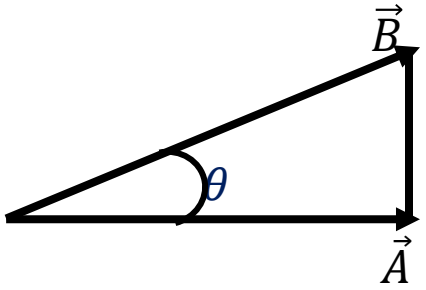
3.4 Cross Product of two vectors:-

The product of multiplication of a vector

$\vec{A} * \vec{B} = \vec{C}$

1) The magnitude of $|\vec{A} * \vec{B}| = |\vec{C}|$ is equal to $|A| . |B| \sin \theta_{(A,B)}$,

$\boldsymbol{\theta}$:- is smaller angler from the first vector to the second vector $(\mathbf{0} \leq \boldsymbol{\theta} \leq \mathbf{180})$



2) The direction of the cross product (i.e. vector c) is particular to the plan formed by **A** and **B**.

3) The sense of the direction is according right hand.

4) $(\vec{A} + \vec{B}) * \vec{C} = \vec{A} * \vec{C} + \vec{B} * \vec{C}$

5)

$$\vec{i} * \vec{i} = 0 \qquad \vec{j} * \vec{j} = 0 \qquad \vec{k} * \vec{k} = 0$$

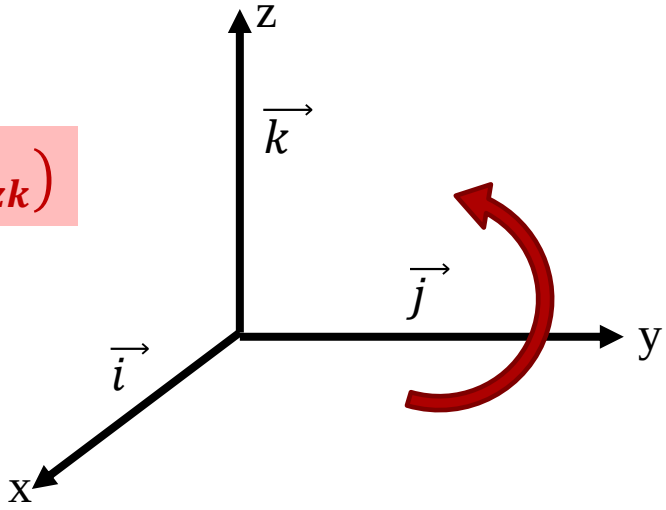
$$\vec{i} * \vec{j} = \vec{k} \qquad \vec{j} * \vec{k} = \vec{i} \qquad \vec{k} * \vec{i} = \vec{j}$$

or

$$\vec{j} * \vec{i} = -\vec{k} \qquad \vec{k} * \vec{j} = -\vec{i} \qquad \vec{i} * \vec{k} = -\vec{j}$$

6)

$$\vec{A} * \vec{B} = \left(\vec{A}_{xi} + \vec{A}_{yj} + \vec{A}_{zk}\right) * \left(\vec{B}_{xi} + \vec{B}_{yj} + \vec{B}_{zk}\right)$$



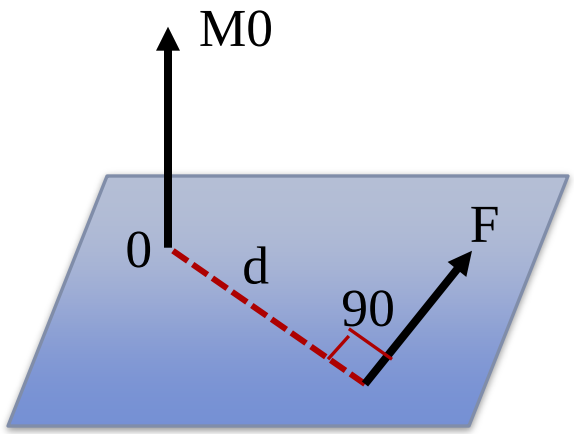
$$\vec{A} * \vec{B} = \begin{vmatrix} +\vec{i} & -\vec{j} & +\vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} =$$

$$\left(A_x B_z - A_z B_y\right)\vec{i} - \left(A_x B_z - A_z B_x\right)\vec{j} + \left(A_x B_y - A_y B_x\right)\vec{k}$$

Moment's in (3D)systems

A) Moment about point.

1. M = F * d (is the magnitude)
2. Direction is ⊥ to the plane formed by the force $\overrightarrow{F}$ and the point 0 (right hand rule).
3. The point 0 is the point of action of the moment.



$\overrightarrow{M_0} = \overrightarrow{r} * \overrightarrow{F_{AB}}$

$\overrightarrow{r}$:- arm of force

$\overrightarrow{r} = \overrightarrow{OA}\ (coordinate\ A - coordinate\ o)\ \ or$

$\overrightarrow{r} = \overrightarrow{OB}\ (\ (coordinate\ B - coordinate\ o$

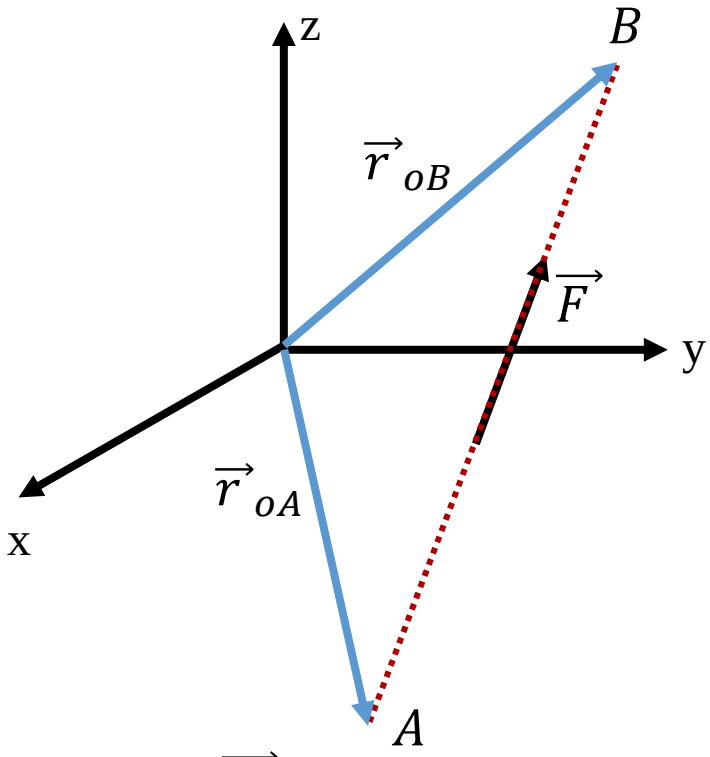
$F = (F_{xi} + F_{yj} + F_{zk})$

$r = (F_{xi} + F_{yj} + F_{zk})$

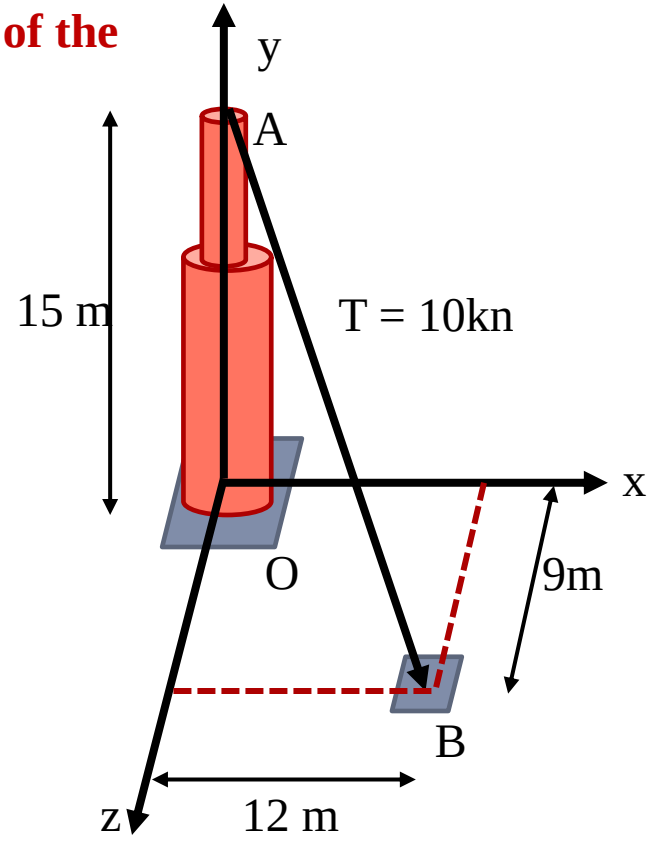
$\overrightarrow{M_0} = \overrightarrow{r} * \overrightarrow{F}$

$$\overrightarrow{M_0} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \boldsymbol{r_x} & \boldsymbol{r_y} & \boldsymbol{r_z} \\ \boldsymbol{f_x} & \boldsymbol{f_y} & \boldsymbol{f_z} \end{vmatrix}$$

$$= (r_y f_z - r_z f_y)\,\vec{i} - (r_x f_z - r_z f_x)\,\vec{j} + (r_x f_y - r_y f_x)\,\vec{k}$$



EX:- the tension in the cable AB is 10Kn, find the moment of the tension a bout point (0)



EX:- Find the moment of the force (400 N) about point (o)